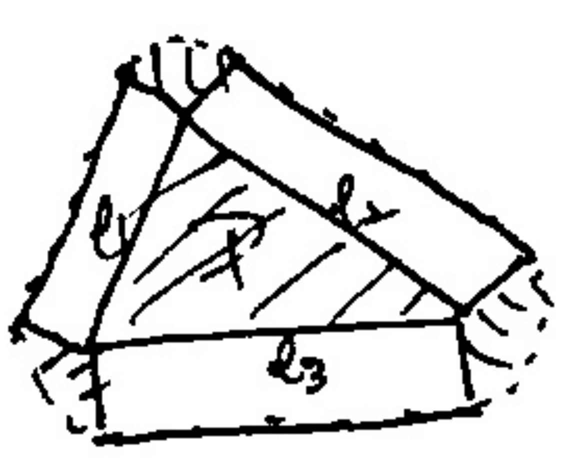
$\mathcal{K}^d$ - $\{$ convex bodies in $\mathbb{R}^d\}$
cpt convex sets

$K+\lambda B$ — λ-thickening of K.

$V(K+\lambda B)$ = polynomial function of λ.

First consider polyhedral.

$$V(P+\lambda B) = \pi\lambda^2 + (\ell_1+\ell_2+\ell_3)\lambda + V(P)$$



$$V(K+\lambda B) = \sum_{i=0}^{\dim K} \mu_i(K)\, c_i\, \lambda^{\dim K - i}$$, where

$\mu_i : \mathcal{K}^d \to \mathbb{R}$ are some functions s.t.

$$\mu(K_1 \cup K_2) = \mu(K_1) + \mu(K_2) - \mu(K_1 \cap K_2)$$
if convex

More generally, $\mu_i : V(\mathcal{K}^d) \to \mathbb{R}$, where

$$V(\mathcal{K}^d) = \{ \textstyle\sum c_j \mathbb{1}_{K_j} \}$$

$$\left.\begin{array}{l} \mu_n(K) = \mathrm{vol}_n(K) \\ \mu_{n-1}(K) = \frac{1}{2}\mathrm{vol}_{n-1}(\partial K) \\ \vdots \\ \mu_1(K) = \text{mean } \cancel{\text{length}} \text{ width} \end{array}\right\}$$ intrinsic volumes - value spectrum

$$\mu_0(K) = 1$$

$\mu_1(\cancel{K}\ B(r)) = 2r$ - diametre.

$$\mu_k(K) = \int_{\mathrm{Gr}(k,\mathbb{R}^d)} \mathbb{1}_{K\cap H}\, dH$$

Hadwiger thm: $\mu_0, \mu_1, \ldots, \mu_d$ form a basis of all valuations on ~~K~~ K^d.

Blaschke worked on this earlier.